Benha University
Faculty Of Engineering at Shoubra

ECE 122
Electrical Circuits (2)(2016/2017)

Lecture (11)
Transient Analysis (P3)

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Reference
Chapter 16

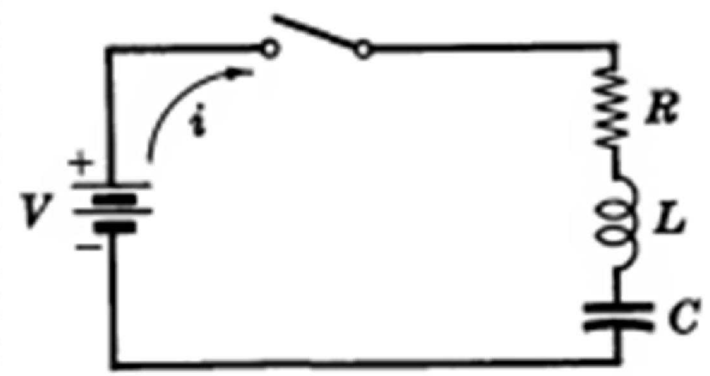
Schaum's Outline Of Theory And Problems Of Electric Circuits

https://archive.org/details/TheoryAndProblemsOfElectricCircuits

Second-Order RLC Transient (Step Response)

- The Switch "S" is closed at t=0
- Applying KVL will produce the following Integro-Differential equation:

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V$$



- Differentiating, we obtain

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0 \quad \text{or} \quad \left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0$$

This second order, linear differential equation is of the homogeneous type with a particular solution of zero.

✓ The complementary function can be one of **three different types according to the roots of the auxiliary equation which** depends upon the relative magnitudes of R, L and C.

$$m^2 + \frac{R}{L}m + \frac{1}{LC} = 0$$

Second-Order RLC Transient (Step Response)

We can Rewrite the auxiliary equation as:

$$m^2 + \frac{R}{L}m + \frac{1}{LC} = 0$$

- The roots of the equation (or natural frequencies):

$$\begin{cases} m_1 = -\dfrac{R}{2L} + \sqrt{\dfrac{R^2}{4L^2} - \dfrac{1}{LC}} \\ m2 = -\dfrac{R}{2L} - \sqrt{\dfrac{R^2}{4L^2} - \dfrac{1}{LC}} \end{cases}$$

Second-Order RLC Transient (Step Response)

Case 1: Overdamped,

$$\frac{R}{2L} > \frac{1}{\sqrt{LC}} \quad \Rightarrow m_1, m_2 \text{ are real and unequal}$$

Natural response is the sum of two decaying exponentials:

$$i_{tr} = K_1 e^{m_1 t} + K_2 e^{m_2 t}$$

Case 2: Critically damped,

$$\frac{R}{2L} = \frac{1}{\sqrt{LC}} \quad \Rightarrow m_1, m_2 \text{ are real and equal.}$$

$$m_1 = m_2 = -\omega_0$$

$$x_c(t) = e^{m_1 t}(B_1 + B_2 t)$$

Use the initial conditions to get the constants

Usually it is reduced to:

$$x_c(t) = B.t.e^{m_1 t}$$

Second-Order RLC Transient (Step Response)

Case 3: Underdamped,

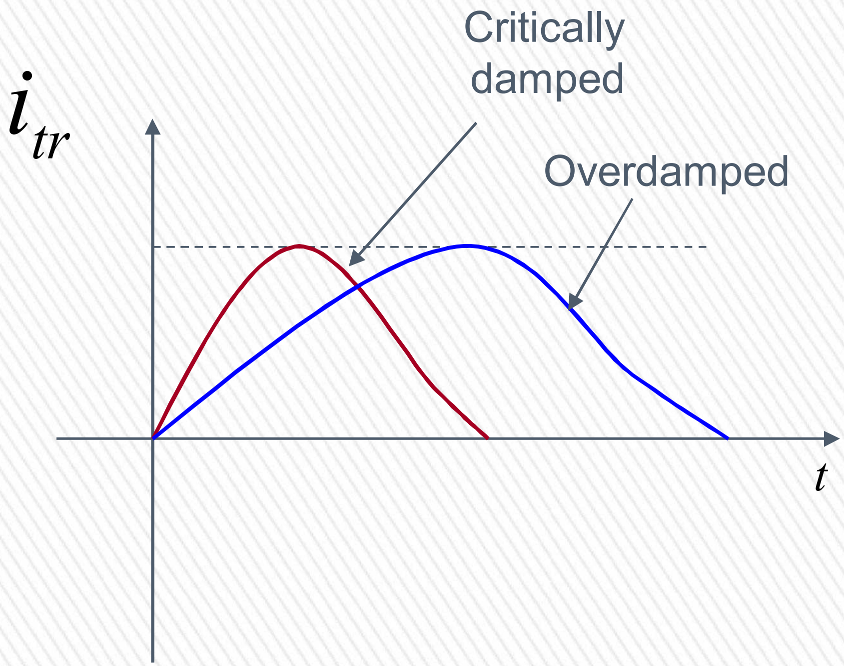
$$\frac{R}{2L} < \frac{1}{\sqrt{LC}}$$

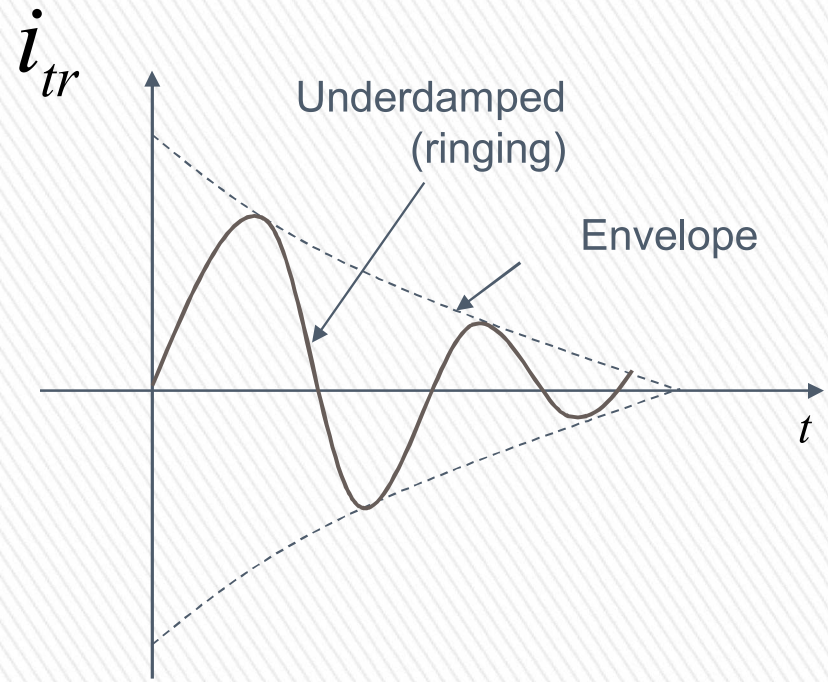
$$\sigma^2 < \omega_o^2 \qquad \Rightarrow \; m_1, m_2 \text{ are complex and conjugate.}$$

$$\sigma = \frac{R}{2L}$$

Natural response is an exponentially damped oscillatory response:

$$i_{tr} = e^{-\sigma t}\{A_1 \cos(\omega_d\, t) + A_2 \sin(\omega_d\, t)\}$$





- ✓ The current in all cases contains the exponential decaying factor (damping factor) assuring that the final value is zero
- ✓ In other words, assuring that the complementary function decays in a relatively short time.

Example

Example

A series RLC circuit with R = 3000 ohms, L = 10 h and C = 200 µf has a constant voltage V = 50 volts applied at t = 0. Find the current transient and the maximum value of the current if the capacitor has no initial charge.

$$50 = Ri + L\frac{di}{dt} + \frac{1}{c}\int i\,dt$$

$$50 = 3000i + 10\frac{di}{dt} + \frac{1}{200\times10^{-6}}\int i\,dt \quad (1)$$

بتفاضل الطرفين

or $(D^2 + 300D + 500)\,i = 0$

بحل المعادلة التربيعية بالقانون العام

$$D = \frac{-B \mp \sqrt{B^2 - 4AC}}{2A} = \alpha \mp \beta$$

$\therefore D_1 = -298.3 \;(\alpha+\beta), \quad D_2 = -1.67 \;(\alpha-\beta)$

Note $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

$$\therefore i = C_1 e^{-1.67t} + C_2 e^{-298.3t} \quad (2)$$

to Find C_1, C_2

at $t = 0, 0^+ \rightarrow i = 0$ sub in 1, 2

Example

∴ $0 = c_1 + c_2$ → (3)

and at $t = 0$ → $50 = 10\, di/dt$ or $di/dt = 5$ → (4)

At $t = 0$ ∴ $di/dt = 5$ ∴ نفاضل المعادلة 2 ونعوض بـ 5

$$5 = (-1.67)\, c_1 e^{-1.67t} - (298.3)\, c_2 e^{-298.3t}$$

$$5 = -1.67\, c_1 - 298.3\, c_2 \rightarrow (6)$$

بحل المعادلات ٣ و 6 نحصل

$c_1 = 0.168$, $c_2 = -0.0168$

∴ $i = 0.0168\, e^{-1.67t} - 0.0168\, e^{-298t}$ مطلوب

to find max current ∴ at $\frac{di}{dt} = 0$ (القيم العظمى)

or $(0.0168)(-1.67)\, e^{-1.67t} - (0.0168)(-298.3)\, e^{-298t} = 0$

→ $t = 0.0175$ sec

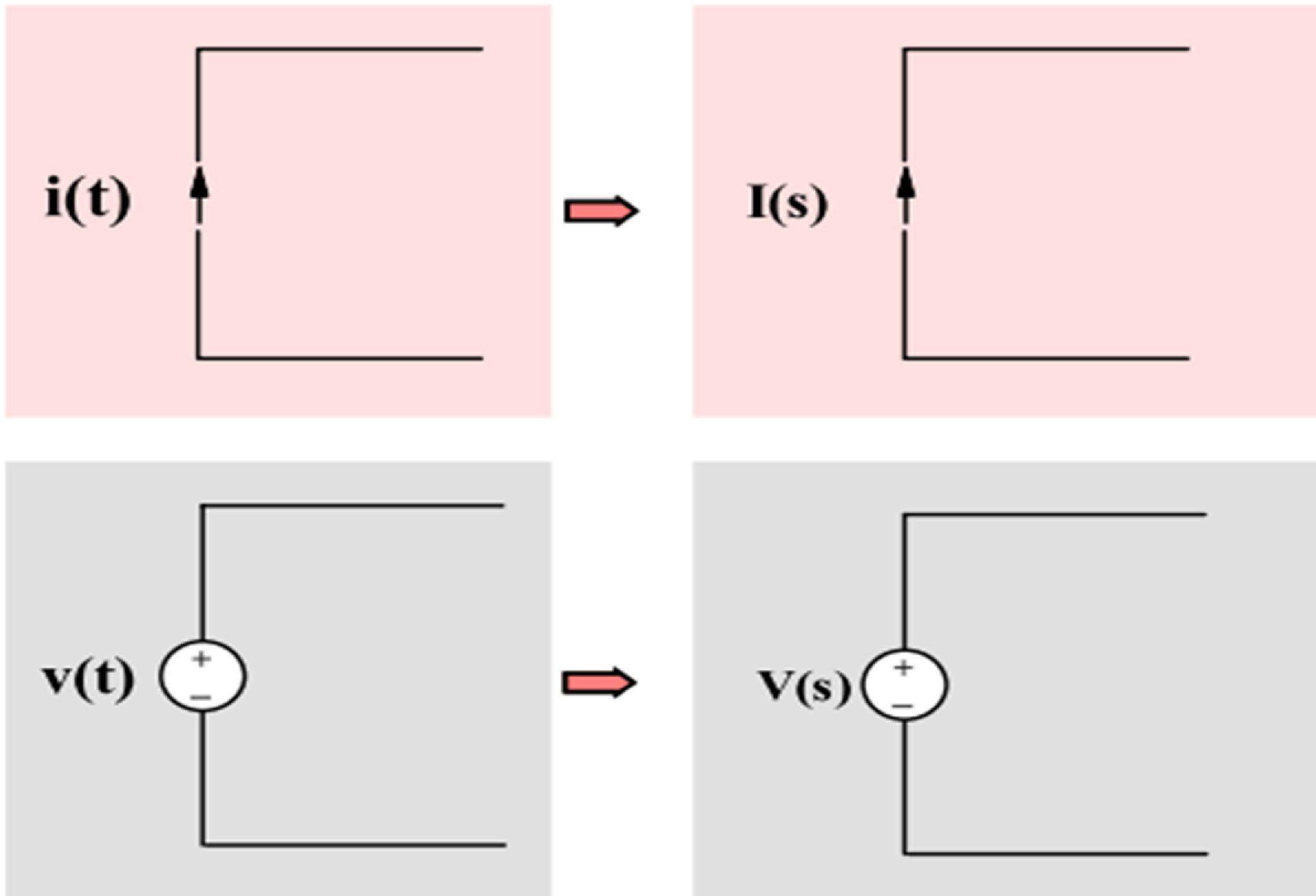
Transient Analysis using Laplace Transform

- Laplace transform is considered one of the most important tools in Electrical Engineering
- It can be used for:

  - ✓ Solving differential equations
  - ✓ Circuit analysis (Transient and general circuit analysis)
  - ✓ Digital Signal processing in Communications and
  - ✓ Digital Control

Transient Analysis using Laplace Transform

Circuit Elements in the "S" Domain



Circuit Elements in the "S" Domain

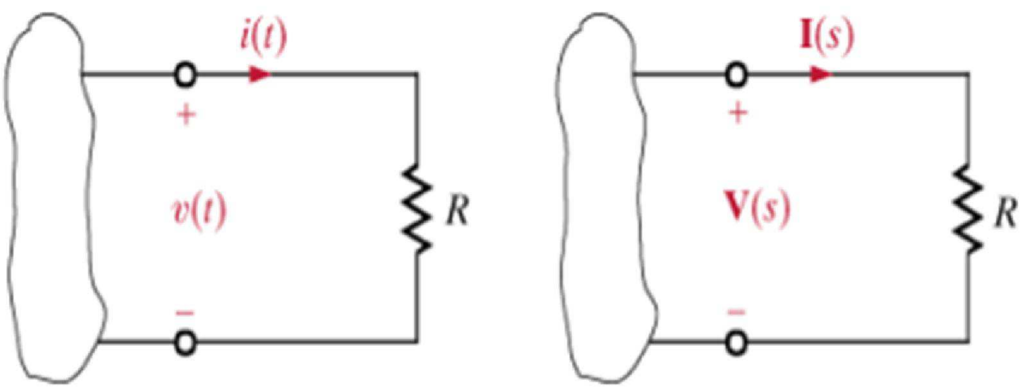
Circuit Element Modeling

The method used so far follows the steps:

1. Write the differential equation model
2. Use Laplace transform to convert the model to an algebraic form

1.0 Resistance

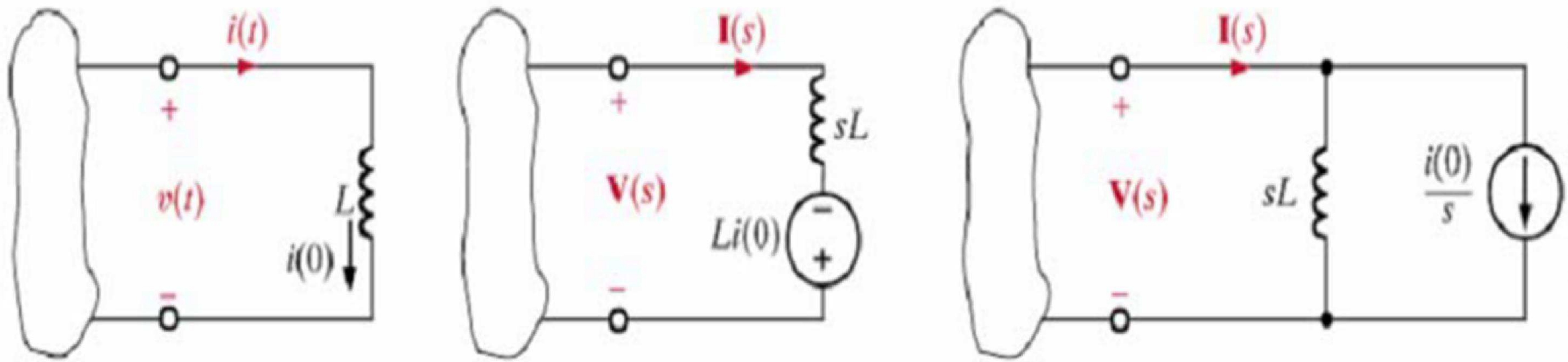
Resistor



$$v(t) = Ri(t) \Rightarrow V(s) = RI(s)$$

Circuit Elements in the "S" Domain

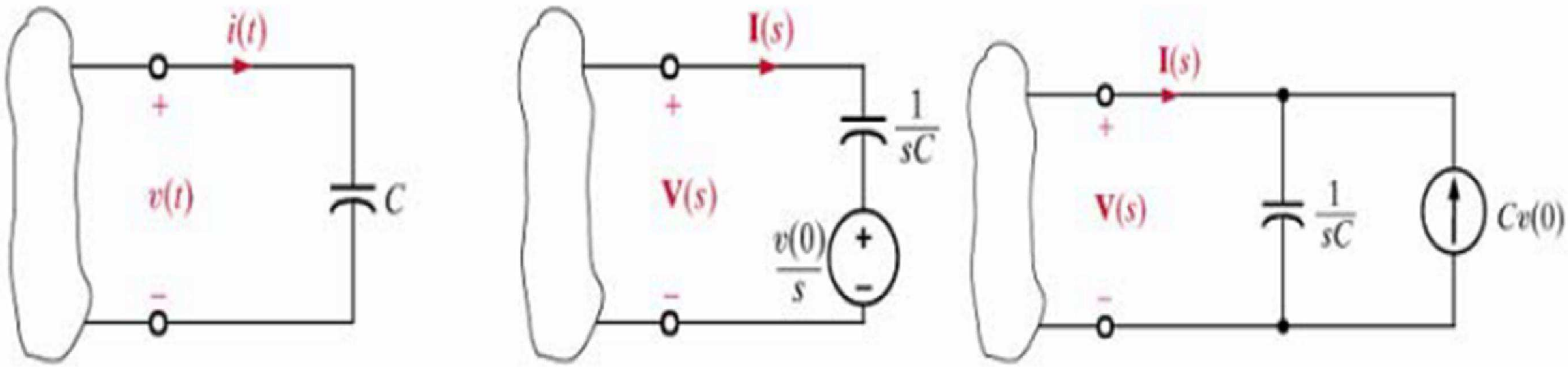
2.0 Inductor



$$v(t) = L\frac{di}{dt}(t) \quad \Rightarrow \quad V(s) = LsI(s) - Li(0) \quad \Rightarrow \quad I(s) = \frac{V(s)}{Ls} + \frac{i(0)}{s}$$

Circuit Elements in the "S" Domain

3.0 Capacitor



$$v_C(t) = \frac{1}{C}\int_0^t i(t)dt + v_C(0)$$

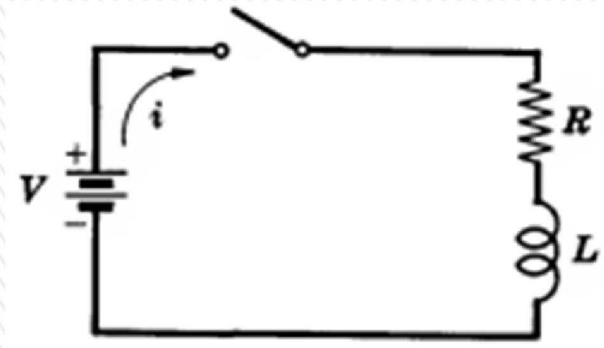
$$V(s) = \frac{1}{Cs}I(s) + \frac{v(0)}{s}$$

$$I(s) = CsV(s) - Cv(0)$$

First-Order RL Transient (Step-Response)

- The switch "S" is closed at t = 0 to allow the step voltage to excite the circuit
- Apply KVL to the circuit in figure:

$$Ri + L\frac{di}{dt} = V$$



Apply Laplace Transform on both sides ➢

$$R.I(s) + L[s.I(s) - i(0)] = \frac{V}{s}$$

i(0) = 0 >> initial value of the current at t = 0

$$I(s).[R + sL] = \frac{V}{s}$$

$$I(s) = \frac{V}{s[R + sL]} = \frac{V/L}{s[s + R/L]}$$

- Apply the inverse Laplace Transform technique to get the expression of the current i(t)

First-Order RL Transient (Step-Response)

- Use the partial fraction technique

$$I(s) = \frac{V/L}{s[s + R/L]} = \frac{A_1}{s} + \frac{A_2}{s + R/L}$$

OR

$$A_1 = \{s * I(s)\}|_{s=0} = \frac{V}{R}$$

$$A_2 = \{(s + R/L) * I(s)\}|_{s=-R/L} = -\frac{V}{R}$$

- Multiply both sides by $s.(s + R/L)$

$$V/L = A_1.(s + R/L) + A_2.s$$

$$\ldots\ldots = (A_1 + A_2).s + A_1.\frac{R}{L}$$

both sides Compare the coefficients

$$A_1 = V/R \qquad A_2 = -V/R$$

- So, the current in s-domain is given by:

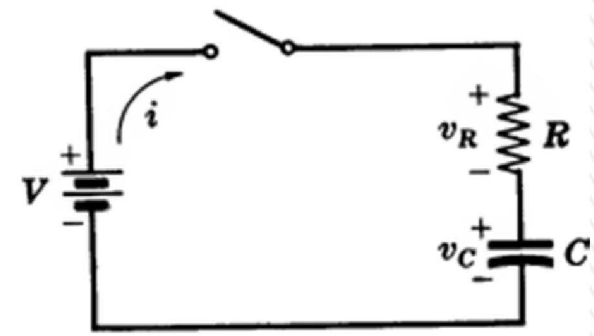
$$I(s) = \frac{V}{R}\left(\frac{1}{s} - \frac{1}{s + R/L}\right)$$

- Apply the inverse Laplace transform :

$$i(t) = \frac{V}{R}(1 - e^{-\frac{R}{L}t}); \; t > 0$$

The same as last lecture

First-Order RC Transient (Step-Response)



- Assume the switch S is closed at t = 0
- Apply KVL to the series RC circuit shown:

$$[\frac{1}{c}\int i(t).dt + v_c(0)] + R.i(t) = V$$

➢ Apply Laplace Transform on both sides

$$[\frac{I(s)}{cs} + \frac{v_c(0)}{s}] + R.I(s) = \frac{V}{s}$$

$V_c(0) = 0$ >> initial value of the voltage at t = 0

$$I(s).[R + \frac{1}{cs}] = \frac{V}{s} \qquad I(s) = \frac{V/s}{[R + 1/cs]} = \frac{V/R}{[s + 1/cR]}$$

➢ Apply the inverse Laplace Transform technique to get the expression of the current i(t)

$$i(t) = \frac{V}{R} e^{-\frac{1}{RC}t} ;\ t > 0$$

The same as last lecture


Thank You